23. $$\int_{-\infty}^{\infty} \frac{dx}{e^x+e^{-x}} = \int_{-\infty}^{0} \frac{1}{e^x+e^{-x}}\,dx + \int_{0}^{\infty} \frac{1}{e^x+e^{-x}}\,dx$$

$$\lim_{b\to\infty}\int_0^b \frac{1}{\left(e^x+\frac{1}{e^x}\right)e^x}\,e^x\,dx = \lim_{b\to\infty}\int_0^b \frac{e^x}{(e^x)^2+1}\,dx$$

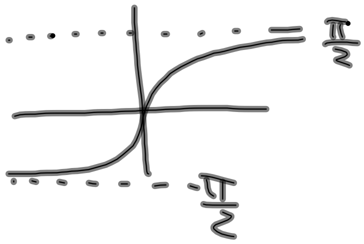
$\int \frac{e^x}{(e^x)^2+1}\,dx$ let $u=e^x$ $du=e^x\,dx$

$$\int \frac{du}{u^2+1} = \tan^{-1}u = \tan^{-1}e^x$$

$$\lim_{b\to\infty} \tan^{-1}e^x \Big|_0^b$$

$$\lim_{b\to\infty} \tan^{-1}e^b - \tan^{-1}e^0$$ half of answer

$$\frac{\pi}{2}-\frac{\pi}{4} = \frac{\pi}{4}$$



Jan 25-9:16 AM

21. $$\int_{-\infty}^{\infty} e^{-|x|}\,dx = \int_{-\infty}^{0} e^{-|x|}\,dx + \int_0^{\infty} e^{-|x|}\,dx$$

$$\int_{-\infty}^{0} e^{--x}\,dx + \int_0^{\infty} e^{-x}\,dx$$

$|x| = -x$
if $x<0$

$$\int_{-\infty}^{0} e^{x}\,dx + \int_0^{\infty} e^{-x}\,dx$$

$x=-2$ $\quad e^{-|-2|} = e^{-2}$

e^{-2}

Jan 25-9:27 AM

15. $$\int_1^{\infty} \frac{5x+6}{x^2+2x}\,dx = \int_1^{\infty} \frac{5x+6}{x(x+2)}\,dx$$

$$= \int_1^{\infty} \frac{A}{x} + \frac{B}{x+2}\,dx \qquad A = \frac{6}{2} = 3$$

$$B = \frac{-4}{-2} = 2$$

$$= \lim_{b\to\infty} \int_1^{b} \frac{3}{x} + \frac{2}{x+2}\,dx$$

$$\lim_{b\to\infty} 3\ln x + 2\ln(x+2) \Big|_1^b$$

$$\lim_{b\to\infty} 3\ln b + 2\ln(b+2) - (3\ln 1 + 2\ln 3) = \infty$$

Jan 25-9:35 AM

8.4b Improper Integrals

Comparison Test P 464

if $0 \le f(x) \le g(x)$

if $\int_a^{\infty} g(x)\,dx$ converges then $\int_a^{\infty} f(x)\,dx$ converges

if $\int_a^{\infty} f(x)\,dx$ diverges then $\int_a^{\infty} g(x)\,dx$ diverges

Jan 17-8:46 PM

$$\int_1^\infty e^{-x^2}\,dx = \lim_{b\to\infty}\int_1^b e^{-x^2}\,dx \le \lim_{b\to\infty}\int_1^b e^{-x}\,dx = \frac{1}{e}$$

$\int e^{-x^2}\,dx$

converges (between 0 & $\frac{1}{e}$)

$\cancel{y = e^{-x^2}}$

$\cancel{y' = e^{-x^2}\cdot(-2x)}$

Jan 17-8:48 PM

Does the integral converge or diverge?

$$\int_1^\infty \frac{dx}{x^5+1} \le \int_1^\infty \frac{dx}{x^5} = \int_1^\infty x^{-5}\,dx$$

$\int_1^\infty \frac{dx}{x^2+1}$

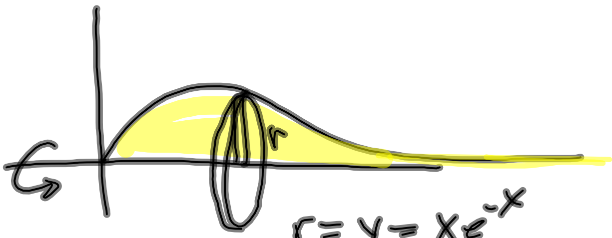
$$\lim_{b\to\infty} \frac{x^{-4}}{-4}\Big|_1^b$$

$$\lim_{b\to\infty} \frac{b^{-4}}{-4} - \frac{1^{-4}}{-4} = \frac{1}{4}$$

$\frac{b^{-4}}{-4} \to 0$

Jan 17-9:00 PM

Find the volume of the solid obtained by revolving the curve about the x-axis $y = xe^{-x}, 0 \le x < \infty$



$r = y = xe^{-x}$

$$\int_0^\infty \pi r^2\,dx = \int_0^\infty \pi (xe^{-x})^2\,dx$$

$$= \int_0^\infty \pi x^2 e^{-2x}\,dx$$

$$= \lim_{b\to\infty} \int_0^b \pi x^2 e^{-2x}\,dx$$

x^2	+	e^{-2x}
$2x$	−	$-\frac{1}{2}e^{-2x}$
2	+	$\frac{1}{4}e^{-2x}$
0		$-\frac{1}{8}e^{-2x}$

$$= \lim_{b\to\infty}\left(-\frac{x^2}{2}e^{-2x} - \frac{2xe^{-2x}}{4} - \frac{2e^{-2x}}{8} \Big|_0^b \right.$$

$$= \lim_{b\to\infty}\left(\frac{-b^2}{2e^{2b}} - \frac{2b}{4e^{2b}} - \frac{2}{8e^{2b}} - \left(0 + 0 - \frac{2}{8}e^0\right)\right)$$

$\downarrow 0 \quad \downarrow 0 \quad \downarrow 0 \qquad \frac{1}{4}$

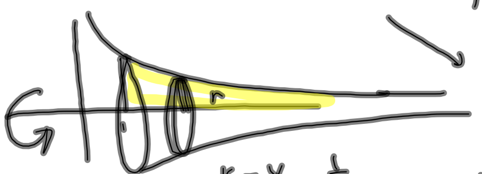
Jan 17-8:49 PM

Gabriel's Horn

Consider the region R in the first quadrant bounded above by y=1/x and on the left by x=1. The region is revolved around the x-axis.

a) Show that R has infinite area. $\int_1^\infty \frac{1}{x}\,dx = \infty$

b) Find the volume of the solid.



$r = y = \frac{1}{x}$

$$\int_1^\infty \pi r^2\,dx = \int_1^\infty \pi \frac{1}{x^2}\,dx$$

$$\lim_{b\to\infty} \pi \int_1^b x^{-2}\,dx$$

$$\lim_{b\to\infty} \pi \frac{x^{-1}}{-1}\Big|_1^b$$

$$\lim_{b\to\infty} \frac{-\pi}{x}\Big|_1^b = \lim_{b\to\infty} \frac{-\pi}{b} - \frac{-\pi}{1} = \pi$$

paradox

Jan 17-8:56 PM